Universe driven by the vacuum of scalar field: V0CDM model

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Abstract

The model of Universe driven by the vacuum fluctuations of scalar fields (gr-qc/0604020, gr-qc/0610148) is compared with both ΛCDM model and deceleration parameter reconstruction from the SN type Ia data.

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Fast progress in accumulating and handling of the astrophysical data about the Universe expansion [1,2,3,4,5] clears the way to testing of different models of the Universe evolution. Although, the ΛCMD model is able to explain the observational data [6], it is necessary to provide a deeper insight into the cosmological constant problem [7,8,9,10,11,12,13,14]. Among numerous approaches to the cosmological constant problem, the quantum field theory (QFT) approach may suggest some solutions.

It is well known that the covariant removing of all divergent terms from the energy-momentum tensor by some regularization procedure leads to the vacuum energy density $\rho_{vac} \sim 1/L^4$, where L is the radius of Universe curvature [17]. This quantity is too small [1] to explain the observed Universe acceleration if one may identify L with the size of a present day Universe.

[1] For the flat expanding Universe and the self-interacting scalar field $V(\phi) \sim \lambda\phi^4$, it is $\rho_{vac} \sim \lambda H^4$ [18], where H is the Hubble constant. This quantity is too tiny even for $\lambda \sim 1$. Nevertheless it was found, that the vacuum energy density can be proportional to the mass of a scalar field squared if the nonperturbative approach to the massive field with $V(\phi) \sim m^2\phi^2/2$ is under consideration. This is the so-called vacuum-cold-dark-matter (VCDM) model [19,20].

On the other hand, the direct ultraviolet momentum (UV) cut-off for evaluation of vacuum energy provides the enormous quantity $\rho_{vac} \sim M_p^4$, where M_p is the Planck mass.

In our previous works, the accelerated expansion of Universe was attributed to the back-reaction of vacuum fluctuations of massless scalar fields (V0CDM model). It was found, that the use of UV cut-off at the Planck level in the equation of motion for the Universe scale factor instead of that in the Friedmann's equation allows explaining the observable value of Universe acceleration. Unlike the model by L. Parker and collaborators [19,20], our model does not require the massive scalar field. In our approach, the effective density of dark energy is proportional to the Hubble constant squared $\rho_{vac} \sim H^2\kappa_{max}^2 \sim H^2 M_p^2$, as it occurs in the holographic dark energy models [21,22,23,24,25,26] (here κ_{max} denotes the UV cut-off of the present day physical momentums).

Below our previous model is summarized and compared with the observed dependence of deceleration parameter from the SN Ia data.

Let us write down the system of Friedman– Lemaître equations for the Universe scale factor a, the density of a matter ρ and the pressure p:

$$
\begin{aligned}
-\frac{1}{2}M_p^2\left(a'^2+\mathcal{K}a^2\right)+\rho a^4+\frac{1}{6}M_p^2\Lambda a^4=0,\\
M_p^2 a''=-M_p^2\mathcal{K}a+(\rho-3p)a^3+\frac{2}{3}M_p^2a^3\Lambda,\\
\rho'+3\frac{a'}{a}(\rho+p)=0,
\end{aligned}
\tag{1}
$$

where the conformal time η implying the metric $ds^2 = a^2(\eta)(d\eta^2+d\sigma^2)$ is used (the reason will be explained below), Λ is the cosmological constant, $\mathcal{K}$ is the signature of space-time and the Plank mass M_p should be read as $M_p = \sqrt{\frac{3}{4\pi G}}$.

The ΛCDM model can be obtained by setting $p = 0$, $\mathcal{K} = 0$ and finally is reduced to the single equation

$$
a'' = 2\frac{a'^2}{a} - \frac{3}{2}a_0\mathcal{H}^2\Omega_m,
\tag{2}
$$

where $a_0 = a(0)$ is the present day scale factor (this moment corresponds to $\eta = 0$ in this article), $\mathcal{H} = \left.\frac{a'}{a}\right|_{\eta=0}$conformal Hubble constant [2] and the constant Ω_m is connected with the matter density $\frac{1}{2}\Omega_m M_p^2\,\mathcal{H}^2 a_0 = \rho\, a^3 = \rho_0 a_0^3$.

Let us remind the V0CDM model [15,16]. The first step is to set $\Lambda = 0$, $p = 0$, $\mathcal{K} = 0$ and add a massless scalar field, which is characterized by the averaged

[2] $\mathcal{H} = H_0 a_0$, where H_0 is the present day Hubble constant.

pressure and the density:

$$\rho_\phi = \frac{1}{V}\int_V \left(\frac{\phi'^2}{2a^2} + \frac{(\boldsymbol{\nabla}\phi)^2}{2a^2}\right) d^3\boldsymbol{r},$$
$$p_\phi = \frac{1}{V}\int_V \left(\frac{\phi'^2}{2a^2} - \frac{(\boldsymbol{\nabla}\phi)^2}{6a^2}\right) d^3\boldsymbol{r}, \tag{3}$$

where V is some volume, which will be set to unity everywhere below. The second step is to turn to the quasiclassical picture, where the scalar field $\hat{\phi}(\eta, \boldsymbol{r})$ is quantum. The resulting base equations for the V0CDM model are

$$-M_p^2\frac{a'^2}{2} + \rho a^4 + \int \left(\frac{a^2 < 0|\hat{\phi}'^2|0 >}{2} + \frac{a^2 < 0|(\boldsymbol{\nabla}\hat{\phi})^2|0 >}{2}\right) d^3\boldsymbol{r} = const,$$
$$M_p^2 a'' = \rho a^3 - \int \left(a < 0|\hat{\phi}'^2|0 > - a < 0|(\boldsymbol{\nabla}\hat{\phi})^2|0 >\right) d^3\boldsymbol{r}, \tag{4}$$
$$\hat{\phi}'' + 2\frac{a'}{a}\hat{\phi}' - \Delta\hat{\phi} = 0,$$

where $< 0|...|0 >$ denotes a mean value over the vacuum state of scalar field. The first equation is the integral of motion for two last equations. However, it should be noted that it is not the Friedman equation because the constant on the right hand side is not zero. The point is that some renormalization is needed to avoid the cosmological constant problem, i.e. huge QFT vacuum energy in the Friedman equation. Instead of determining the renormalization constant, one can consider two last equation and fix the constant assigning the initial condition for the equations. It is very important, that in conformal time a renormalization is not required for the second equation. The reason is that the equation contains exact difference of the kinetic and potential energies of the field oscillations. In the Minkowski space-time this difference is exactly zero by virtue of the virial theorem for an oscillator, which states that the kinetic energy is equal to the potential one in the virial equilibrium. In the expanding Universe this difference is proportional to the Hubble constant squared.

Scalar field can be decomposed in a complete set of the modes $\phi(\boldsymbol{r}) = \sum_{\boldsymbol{k}} \phi_{\boldsymbol{k}} e^{i\boldsymbol{k}\boldsymbol{r}}$ and quantization of the modes consists in postulating [17]

$$\hat{\phi}_{\boldsymbol{k}} = \hat{\mathrm{a}}^+_{-\boldsymbol{k}}\chi^*_k(\eta) + \hat{\mathrm{a}}_{\boldsymbol{k}}\chi_k(\eta), \tag{5}$$

where complex functions $\chi_k(\eta)$ satisfy the relations:

$$\chi''_k + k^2\chi_k + 2\frac{a'}{a}{\chi'}_k = 0,$$
$$a^2(\eta)(\chi_k\,{\chi'_k}^* - \chi^*_k\,\chi'_k) = i. \tag{6}$$

The adiabatic approximation

$$\chi_k(\eta) = \frac{\exp\left(-i\int_0^\eta \sqrt{k^2 - \frac{a''(\tau)}{a(\tau)}}\, d\tau\right)}{\sqrt{2}a(\eta)\sqrt[4]{k^2 - \frac{a''(\eta)}{a(\eta)}}} \tag{7}$$

allows calculating the difference of the kinetic and potential energies of field oscillators up to the second-order terms:

$$\int \left(a < 0|\hat{\phi}'^2|0 > -a < 0|(\boldsymbol{\nabla}\hat{\phi})^2|0 >\right) d^3\boldsymbol{r} =$$
$$\sum_{\boldsymbol{k}} a < 0|\hat{\phi}'_{\boldsymbol{k}}\hat{\phi}'_{-\boldsymbol{k}}|0 > -k^2 a < 0|\hat{\phi}_{\boldsymbol{k}}\hat{\phi}_{-\boldsymbol{k}}|0 >= \sum_{\boldsymbol{k}} a({\chi'_k}^*\chi'_k - k^2\chi_k^*\chi_k) \approx$$
$$\frac{1}{2}\left(-\frac{a''}{a^2} + \frac{a'^2}{a^3}\right)\sum_{\boldsymbol{k}}\frac{1}{k} + O(a'^3) + O(a'a'') + O(a'''), \tag{8}$$

where we imply that a' is the first-order quantity, a'' is the second-order one, a''' is the third-order one and so on.

Using (8) in (4) leads to the master equation of V0CDM model in the form:

$$M_p^2 a'' = \frac{1}{2}\Omega_m M_p^2 \mathcal{H}^2 a_0 + \frac{1}{2}\left(\frac{a''}{a^2} - \frac{a'^2}{a^3}\right)\sum_{\boldsymbol{k}}\frac{1}{k}. \tag{9}$$

As it was shown in Refs. [15,16], ultraviolet (UV) cut-off of the present day physical momentums k/a_0 in the sum $\sum_{\boldsymbol{k}}\frac{1}{k}$ at the Planck level $\kappa_{max} = k_{max}/a_0 \sim M_p$ can explain the observed value of Universe acceleration. In principle, the exact value of UV cut-off has to result from the Planck scale physics. But, in absence of such a result, we can extract it from the observed value of the Universe acceleration at a particular time. Equivalently, in order to describe the further Universe evolution, one can do the following trick:

differentiation of Eq. (9) gives

$$M_p^2 a''' = \left(\frac{3a'^3}{2a^4} - \frac{2a''a'}{a^3} + \frac{a'''}{2a^2}\right)\sum_{\boldsymbol{k}}\frac{1}{k}, \tag{10}$$

and exclusion of $\sum_{\boldsymbol{k}}\frac{1}{k}$ from Eqs. (9), (10) results in the final V0CDM equation

$$a''' = \frac{a'\left(a_0\mathcal{H}^2\Omega_m - 2a''\right)\left(4aa'' - 3a'^2\right)}{a\left(a_0\mathcal{H}^2\Omega_m a - 2a'^2\right)}. \tag{11}$$

A validity range of Eq. (11) is defined by the next terms in the expansion (8). According to Refs. [15,16], the next terms contain additional multiplier

$a'/(ak_{max})$ as compared with the main term, where k_{max} is the UV cut-off $k_{max}/a_0 \sim M_p$ [15,16]. Thus Eq. (11) is valid if $\frac{a'}{a} \ll M_p a_0$, or $\dot{a} \ll M_p a_0$. Certainly, at early stage of the Universe evolution the master equation has to be supplemented with the relativistic matter term.

The next step is to solve Eq. (11) numerically and to find $a(\eta)$. Then inverting the equality $z(\eta) = \frac{a_0}{a(\eta)} + 1$ results in the dependence $\eta(z)$ and finally in the deceleration parameter

$$q(z) = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{a''(\eta(z))a(\eta(z))}{a'^2(\eta(z))} + 1, \tag{12}$$

where dot means differentiation over the cosmic time t.

It is interesting to compare results of the V0CDM model with that of the ΛCDM model and with the reconstruction of the deceleration parameter from the SN type Ia data. It seems natural to take some neutral reconstruction, which does not assume a particular model of dark energy or gravity. For instance, it has be made in Ref. [27], where the parametrization $q(z) = \frac{1}{2} + \frac{q_1 z + q_2}{(1+z)^2}$ is used. One can see from Fig. 1, that the V0CDM curve as well as that of ΛCDM can be put within the 1σ-error channel. The parameter Ω_m amounts 0.27 for both models and the initial condition is $\frac{a''(0)a_0}{a'^2(0)} = 1.8$ for the V0CDMmodel [3]. These values are chosen to fit the curves within a thin waist of the experimental data channel near z=0.2.

Concerning a reconstruction of the deceleration parameter from the recent "gold sample" data (shown in Fig. 2), both models fail to hit the 1σ-error channel and the parameter variation, namely Ω_m for ΛCDM and $a''(0)$ for VCDM, does not provide a better fitting. Certainly, the extended ΛCDM model, assuming some evolution of the equation of state $w(z)$ instead of constant $w = -1$ allows improving the agreement with observations. The V0CDM model has no such a free parameter. Moreover, treating of the V0CDM model in terms of the equation of state

$$w(z) = \frac{2q - 1}{3\,(1 - \Omega_m \mathcal{H}^2 a_0 a/a'^2)} \tag{13}$$

is meaningless (see. Fig. 3), because the V0CDM Friedman equation is valid only up to some constant.

To summarize, we have considered the V0CDM model offered in our previous works [15,16]. In this model, the Universe acceleration results from the vacuum fluctuations of fundamental scalar fields [4] .

[3] It should be reminded that this quantity is proportional to the UV cut-off of momentums [15,16] and will result from a future Planck scale physics.

[4] According to [16], there are at least there fundamental scalar fields: one of the

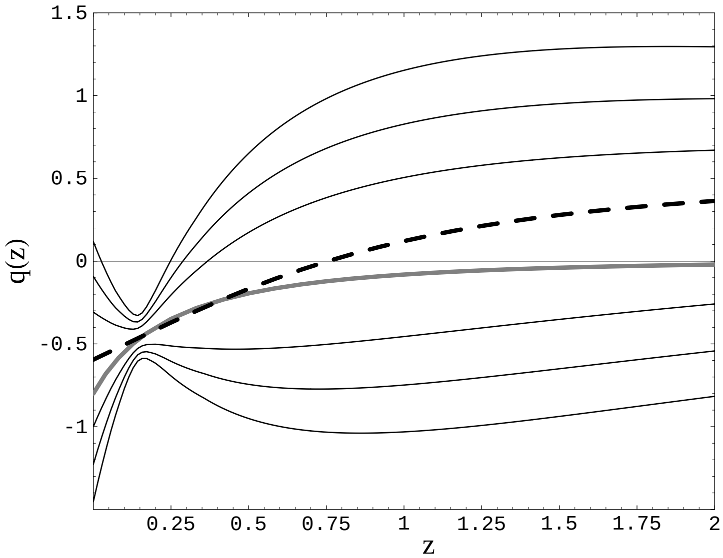

Fig. 1. V0CDM curve (bold grey) of the acceleration parameter evolution and that of ΛCDM (dashed) put on the 1σ, 2σ, 3σ error channels (thin lines) of the reconstruction [27] from the 115 SN Ia data.

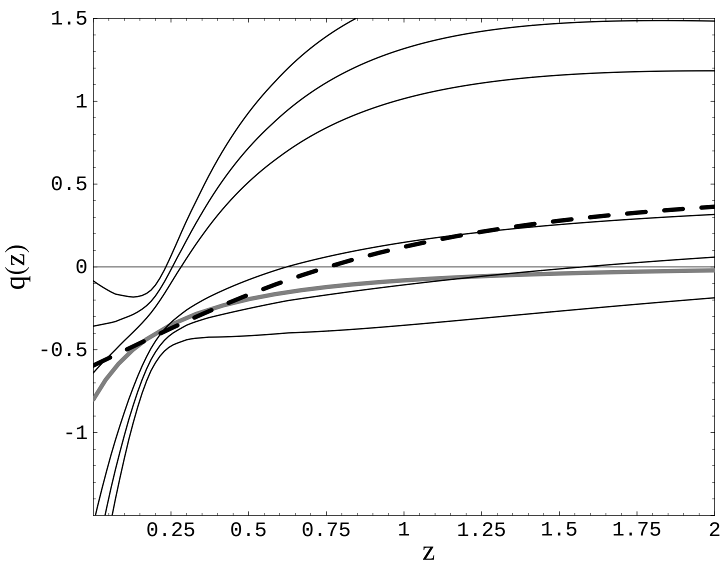

Fig. 2. V0CDM curve (bold grey) of the acceleration parameter evolution and that of ΛCDM (dashed) put on the 1σ, 2σ, 3σ error channels (thin lines) of the reconstruction [27] from the 157 gold sample SN Ia data.

It is shown that both V0CDM and ΛCDM model- fall into the 1σ error channel of the deceleration parameter reconstruction [27] from the 115 supernova Ia sample of data, whereas this does not occur for the 157 gold sample data.

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standard model and two degrees of freedom of the tensor gravitational wave, which are the equivalents of two scalar fields.

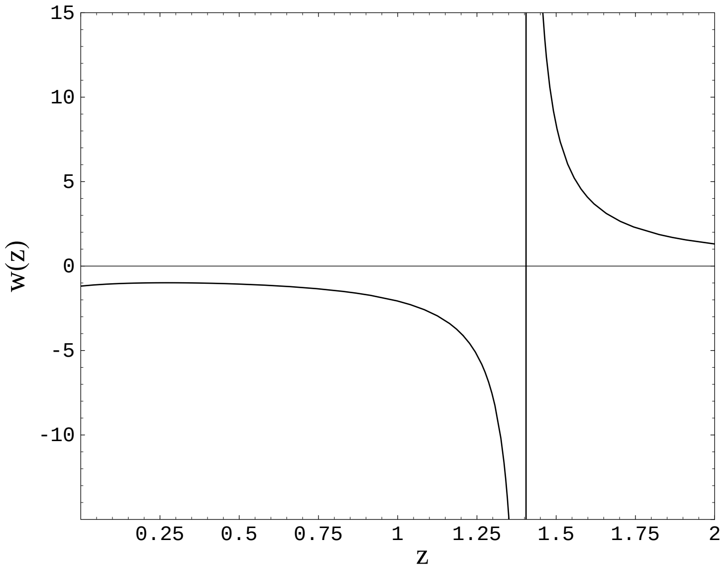

Fig. 3. Effective equation of state $w(z) = p/\rho$ for the V0CDM model calculated using Eq. (13). It should be noted, that $w(z)$ is singular.

The calculation shows that the V0CDM curve depends on Ω_m weaker than it takes a place in the ΛCDM case. Another difference between the models is that V0CDM does not predict the change from acceleration to deceleration within $0 < z < 2$. If the father observations will insist on such a change within this region, some modification of V0CDM should be required, because it has no tuning parameters. Some possibility of such a modification is a theory based on the truncation of physical momentums $k/a(\eta) \sim M_p$ rather than that of static momentums $k \sim a_0 M_p$. This would require the consideration in a system of reference in which Universe looks like the Hoyle-Narlikar one [28,29].

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